

The Teenage Mutant Ninja Turtles are throwing a party, a pizza party, of course. For Problems 1-8, consider the Preference Schedule below, representing votes on which kind of pizza to get for the party. The choices are: **Pepperoni**, **Sausage**, **Mushroom**, and **Veggie**.

Votes	2	2	1	1	1
1st	P	S	P	M	V
2nd	S	M	M	V	M
3rd	M	P	V	S	P
4th	V	V	S	P	S

- (1) How many votes were cast?

7

- (2) Using Plurality, which pizza wins the election? How many votes did it get?

Pepperoni - 3 votes

- (3) Using the Borda Count Method, how many points did each pizza receive? What wins the election if the Borda Count is used?

$$P: 4(2+1) + 2(2+1) + 1(1) = 19$$

$$S: 4(2) + 3(2) + 2(1) + 1(1+1) = 18$$

$$M: 4(1) + 3(2+1+1) + 2(2) = 20$$

$$V: 4(1) + 3(1) + 2(1) + 1(2+2) = 13$$

Mushroom wins

- (4) How many votes are needed for a majority?

$$\frac{7}{2} = 3.5$$

4 votes

- (5) Using Plurality with Elimination, which pizza gets eliminated first, if any? What wins the election if Plurality with Elimination is used?

eliminate M & V first

then Pepperoni wins

- (6) Using Pairwise Comparison, how many comparisons must be made?

$$\frac{N(N-1)}{2} = \frac{4(3)}{2} = \boxed{6}$$

Votes	2	2	1	1	1
1st	P	S	P	M	V
2nd	S	M	M	V	M
3rd	M	P	V	S	P
4th	V	V	S	P	S

(7) Using Pairwise Comparison, complete the table of comparisons, and determine the winner.

Pairings	Votes	Winner/Points
P v. S	P(4) S(3)	P - 1
P v. M	P(3) M(4)	M - 1
P v. V	P(5) V(2)	P - 1
S v. M	S(4) M(3)	S - 1
S v. V	S(4) V(3)	S - 1
M v. V	M(6) V(1)	M - 1

3 way tie  
between P, M, & S

(8) Using Pairwise Comparison, is there a Condorcet Candidate? If so, what candidate?

No (no candidate has  $N-1=3$  points)

The Turtles aren't ordering a Veggie Pizza-this is a party! For Problems 9-12, consider the Preference Schedule below, representing the votes after Veggie Pizza is removed from the election.

Votes	2	2	1	1	1
1st	P	S	P	M	M
2nd	S	M	M	S	P
3rd	M	P	S	P	S

(9) Using Plurality, which pizza wins the election? How many votes did the winner get?

Pepperoni - 3 votes

(10) Using the Borda Count Method, how many points did each pizza receive? Which won the election if the Borda Count is used?

$$P: 3(2+1) + 2(1) + 1(2+1) = 14$$

$$S: 3(2) + 2(2+1) + 1(1+1) = 14$$

$$M: 3(1+1) + 2(2+1) + 1(2) = 14$$

3 way tie  
between P, S & M

(11) Using Pairwise Comparison, complete the table of comparisons, and determine the winner.

Pairings	Votes	Winner/Points
P vs. S	P(4) S(3)	P - 1
P vs M	P(3) M(4)	M - 1
M vs S	M(3) S(4)	S - 1

3 way tie  
between  
P, S, & M

(12) Using Pairwise Comparison, is there a Condorcet Candidate? If so, what candidate?

No (No candidate got 2 points)

The Turtles and Splinter want to toast their friends at the party and thank them for coming, but only Donatello and Leonardo are willing to speak. For Problems 13-15, consider the Ballots below, representing the votes on who should do the toasting. Choices were: Donatello and Leonardo.

	BALLOT	BALLOT	BALLOT	BALLOT	BALLOT
1st	D	D	L	D	L
2nd	L	L	D	L	D

(13) In this election, who wins Plurality?

Donatello

(14) Assume Splinter is a dictator.

(a) If Splinter casts Ballot 4, who wins?

Donatello

(b) If Splinter casts Ballot 5, who wins?

Leonardo

(15) Decide which 2-candidate fairness idea is violated if the following happens. Answers could be: "Anonymous", "Monotone", "Neutral", or "None".

(a) Michelangelo and Raphael switch ballots before casting them, and this does not change the outcome of the election.

None

(b) Michelangelo and Raphael switch ballots before casting them, and this changes the outcome of the election.

Anonymous

(c) Donatello has the 3rd ballot, and changes it to vote for himself, but now he loses.

Monotone

(d) Donatello has the 3rd ballot, and changes it to vote for himself, and he still wins.

None

(e) Distracted by the smell of pizza, all of the votes were entered backwards, meaning a 2nd place vote was really a 1st place votes, and vice versa. Correcting the votes does not change the outcome.

Neutral

(f) Distracted by the smell of pizza, all of the votes were entered backwards, meaning a 2nd place vote was really a 1st place votes, and vice versa. Correcting the votes does change the outcome.

None

(16) Do any Voting Method examples shown above in Problems 1-12 violate the Majority Criterion? If so, state the problem number where this occurred.

[No] - we didn't have a Majority Candidate in these examples, and thus can't violate the Majority Criterion.

(17) Fill in the blanks. Possible choices are listed in the box below. You may use a word more than once.

Majority Criterion	Condorcet Criterion	Monotonicity Criterion
Independence of Irrelevant Alternatives Criterion		
Anonymous	Monotone	Neutral
2 candidate	3 or more candidate	
Majority Rule	Minority Rule	Dictatorship
		Imposed Rule

- (a) May's Theorem states that in a 2 candidate election, Majority Rule is the only voting system that is Anonymous, Monotone, and Neutral. This means that Majority Rule is the best form of 2 candidate voting.
- (b) In a 2 candidate election, a voting system is Neutral if it treats candidates equally. This means that if every voter switched their vote to the other candidate, the outcome of the election switches too.
- (c) In a 2 candidate election, a voting system is Monotone if it is impossible for a winning candidate to become a losing candidate by gaining votes or for a losing candidate to become a winning candidate by losing votes.
- (d) In a 2 candidate election, a voting system is Anonymous if it treats all of the voters equally. If 2 voters trade ballots, the outcome of the election is unchanged.
- (e) We have a theorem that states: If a voting system satisfies the Condorcet Criterion, then the majority candidate is the winner, and thus the Majority Criterion is satisfied also.
- (f) In a 3+ candidate election, a voting system satisfies the Condorcet Criterion if the Condorcet Candidate always wins.
- (g) In a 3+ candidate election, a voting system satisfies the Majority Criterion if a candidate with over 50% of first-place votes automatically wins the election.
- (h) In a 3+ candidate election, a voting system satisfies the Independence of Irrelevant Alternatives Criterion if a candidate wins a first election and then one of the losing candidates drops out before the second election, then the original winner wins the second election.
- (i) In a 3+ candidate election, a voting system satisfies the Monotonicity Criterion if an improvement in a given candidate's vote, without changing the relative quality of the other candidates' votes, does not hurt the given candidate's chance of winning the election.
- (j) Arrow's Impossibility Theorem states that in a 3 or more candidate election, there is no fair method of voting that will simultaneously satisfy the Majority Criterion, Condorcet Criterion, Monotonicity Criterion, and IIA Criterion.